

Ground states of the Josephson vortex lattice in layered superconductors

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We consider the ground state configurations of the Josephson vortex lattice in layered superconductors. Due to commensurability effects with the layered structure, the lattice has multiple configurations, both aligned with layers and rotated at finite angle. At low fields the lattice switches between these configurations via first-order phase transitions. These transitions become more frequent at smaller fields. With increasing magnetic field a dilute lattice transforms first into a sheared dense lattice. With further increase of field, the shear deformation smoothly vanishes at a second-order phase transition.

A magnetic field applied parallel to the layers generates a lattice of Josephson vortices with many unusual static and dynamic properties. An important field scale is set by the anisotropy factor γ and interlayer periodicity s , $B_{cr} = \Phi_0/(2\pi\gamma s^2)$ (in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8-\delta}$, $B_{cr} \sim 0.3 - 0.5\text{T}$). There are two very different regimes depending on the strength of the magnetic field. In the dilute lattice regime, $B_x \ll B_{cr}$, the nonlinear cores of Josephson vortices are well separated and the distribution of currents and fields is very similar to that in continuous anisotropic superconductors. At high fields, $B_x > B_{cr}$, the dense lattice regime is realized where the cores of the Josephson vortices overlap. In this regime the Josephson vortices fill all layers homogeneously¹. This state is characterized by very weak modulations of the in-plane and Josephson currents. In this proceedings we consider the evolution of the ground state configuration with increasing magnetic field.

As the centers of the Josephson vortices must be located between the layers, the layered structure plays crucial role in selection of the ground-state lattice configurations. The Josephson vortex lattice is commensurate with the layered structure only at the discrete set of magnetic fields. At small magnetic fields, $B_x < \Phi_0/(2\pi\gamma s^2)$, the Josephson vortex lattice can be described by anisotropic London model (see, e.g., Refs. 2,3). This model has a simple scaling property: in scaled coordinates, $\tilde{z} = z/s$, $\tilde{y} = y/\gamma s$, the energy becomes isotropic in \tilde{z} - \tilde{y} plane, which means that the ground state in these coordinates corresponds to an ideal triangular lattice which is degenerate with respect to rotations in this plane. In real coordinates these rotations correspond to “elliptic rotations”. As a consequence, the family of commensurate lattices includes lattices aligned with the layers, as well as misaligned ones. To make a full classification of commensurate lattices we consider a general lattice shown in Fig. 1a^{2,3}. The lattice is characterized by three parameters: in-plane period a , distance between vortex rows in c direction $b = Ns$, and relative shift between the neighboring vortex rows in c direction qa . The lattice shape is characterized by the two dimensionless parameters, q and ratio $r = b/a$. The lattice parameters are related to the in-plane magnetic field, B_x , as $B_x = \Phi_0/(ab)$. Structures aligned with the layers correspond to $q = 1/2$. As the replacement $q \rightarrow 1 - q$

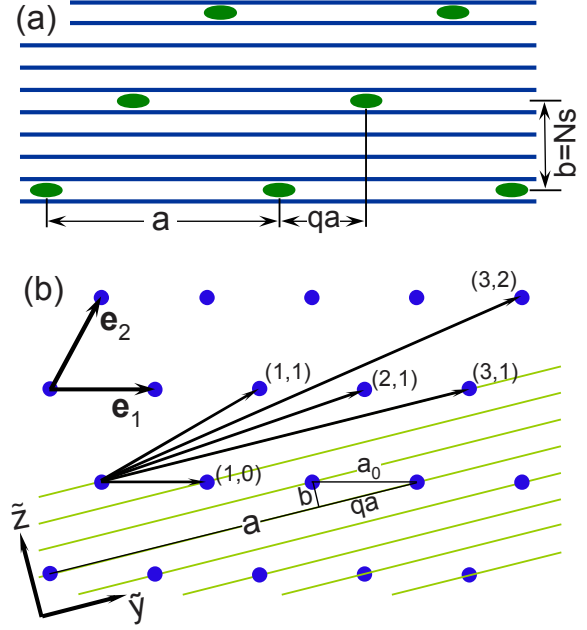


FIG. 1: (a) General Josephson vortex lattice and its parameters. (b) Orientation of layered structure with respect to ideal lattice (in scaled coordinates). Layered structure fits the ideal lattice only if it is oriented along one of the crystallographic directions, which is characterized by two numbers (n, m) , corresponding to expansion of the direction vector over the two basic lattice vectors, \mathbf{e}_1 and \mathbf{e}_2 . Several possible directions are shown with the corresponding indices (n, m) . The lower part of the figure illustrates the lattice parameters, a , b , and q for one possible orientation.

corresponds to the mirror reflection with respect to x - z plane, every structure with $q \neq 1/2$ is double-degenerate.

We start with classification of the lattices exactly commensurate with the layered structure giving the set of commensurate fields (see also Ref. 4). The analysis of commensurability conditions can be done most conveniently in scaled coordinates (\tilde{y}, \tilde{z}) . In these coordinates the ground-state configuration corresponds to regular triangular lattice with period $a_0 = \sqrt{2\Phi_0/\sqrt{3}\gamma s^2 B_x}$. It is convenient to consider orientation of the layered structure with respect to this lattice rather than the other way round. The layered structure fits this lattice only if

it runs along one of the crystallographic directions, see Fig. 1b. The direction (n, m) is defined by the lattice vector, $\mathbf{e}_{n,m}$, which can be expanded over the two basic lattice vectors, $\mathbf{e}_{(n,m)} = n\mathbf{e}_1 + m\mathbf{e}_2$. For nonequivalent directions n and m must be relatively prime numbers. In particular, two aligned configurations correspond to $(n, m) = (1, 0)$ and $(1, 1)$. Any such direction corresponds to set of matching fields, $B_{(n,m)}(N)$, which can be found by direct geometrical calculation⁴

$$B_{(n,m)}(N) = \frac{\sqrt{3}}{2} \frac{\Phi_0}{N^2 \gamma s^2 (n^2 + nm + m^2)}. \quad (1)$$

This result essentially rely on the London approximation, which implies a very strong inequality $N\sqrt{n^2 + nm + m^2} \gg 1$. Number of vortex-free layers per unit cell is given by $N - 1$. The case $N = 1$ represents a special situation when all the layers are filled with vortices and equivalent. It is interesting to note that even for dilute lattice one can have the Josephson vortices in every layer ($N = 1$) in the case of high-order commensurability ($n, m \gg 1$). In ideal situation the lattice switches between different commensurate configuration via series of first-order phase transitions. Number of competing states rapidly increases with field decrease. In addition to giving general ground state, these lattices describe multiple metastable states with unique hierarchical properties studied in Refs. 2,3.

Full analysis of the structure evolution requires energy consideration. The London model does not describe layered superconductor at high fields. To obtain lattice structures in this region one has to consider more general Lawrence-Doniach model. The transition between the *aligned* lattices have been studied within this model by Ichioka⁵. However, our analysis shows that at many fields the true ground state is not given by aligned lattice. If we limit ourself only to aligned configurations, we reproduce results of Ref. 5 at high anisotropies.

Within the Lawrence-Doniach model, at fields $B_x \gg \Phi_0/(4\pi\lambda\lambda_c)$ the lattice energy can be represented as

$$f_{J1} = \frac{B_x^2}{8\pi} + \frac{B_x\Phi_0}{(4\pi)^2\lambda\lambda_c} u(N, q, h) \quad (2)$$

where $\lambda \equiv \lambda_{ab}$ and λ_c are the components of the London penetration depth, $h \equiv 2\pi\gamma s^2 B_x/\Phi_0$ is the reduced magnetic field, and the reduced energy $u(N, q, h)$ is given by

$$u(N, q, h) = \frac{1}{\pi} \sum_{n=1}^N \int d\tilde{y} \left(\frac{1}{2} \left(\frac{d\phi_n}{d\tilde{y}} \right)^2 + 1 - \cos(\phi_{n+1} - \phi_n - h\tilde{y}) \right) \quad (3)$$

To match the London limit³, we write $u(N, q, h)$ in the form

$$u(N, q, h) = \frac{1}{2} \ln \frac{1}{h} + 1.432 + G(N, q, h) \quad (4)$$

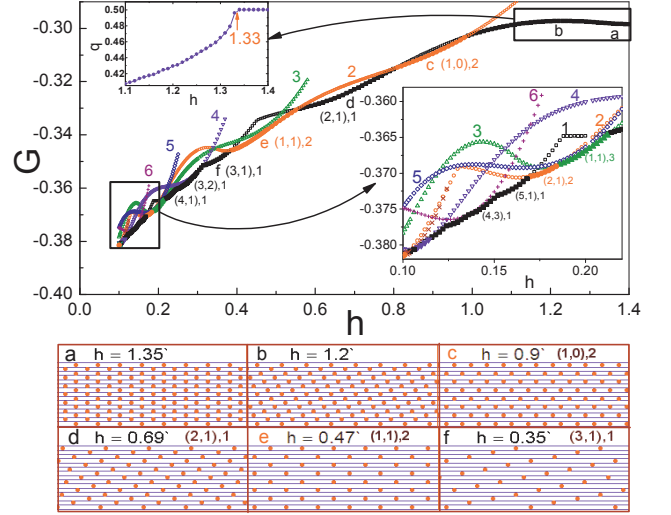


FIG. 2: The field dependence of the energy function $G(N, q, h)$ for the ground state and competing states. Each branch corresponds to the minimum of $G(N, q, h)$ with respect to q at fixed h and N . The curves are marked by value of N . Lattice configurations in scaled coordinates are shown at six marked fields. The left inset shows the field dependence of the shift parameter q near the structural transition of the dense lattice. The right inset illustrates competition between different configuration at smaller fields. The low-field local minima are marked by the indices corresponding to commensurate configurations in the format $(n, m), N$.

where the function $G(N, q, h)$ defined by this equation approaches the London limit³, $G_L(r = N^2 h/(2\pi), q)$, for $h \rightarrow 0$ with

$$G_L(r, q) = \frac{\pi r}{6} + \sum_{l=1}^{\infty} \frac{1}{l} \frac{\exp(-2\pi r l) + \cos(2\pi q l)}{\cosh(2\pi r l) - \cos(2\pi q l)} - \frac{1}{2} \ln(2\pi r).$$

This function depends only on lattice shape. Its absolute minimum corresponding to the triangular lattice is given by $G_L(\sqrt{3}/2, 1/2) = -0.4022$.

We explore the evolution of the ground-state configuration by direct numerical minimization of the energy (3) with respect to lattice parameter N and q defined in Fig. 1, i.e., we computed reduced ground-state energy defined as $G(h) \equiv \min_{N,q}[G(N, q, h)]$. The field dependence of the energy function $G(N, q, h)$ is shown in Fig. 2 for the ground state and competing states. Each curve corresponds to the minimum of $G(N, q, h)$ with respect to q at fixed h and N and it is marked by the value of N . Every branch crossing corresponds to a first order phase transition between different commensurate states. Below, we show the first six lattice configurations which are realized with field decrease. At low fields we specify for each of these ground-state configurations corresponding indices (n, m) and period N in the format $(n, m), N$. At small fields the local minima of branches occur at commensurate fields corresponding to (1).

Let us review the lattice evolution with decreasing

magnetic field. At high fields an aligned dense Josephson vortex lattice is realized (structure *a*). This lattice becomes unstable at $h \approx 1.33$. Below this value, shear deformation develops corresponding to decrease of q below $1/2$ (see structure *b*). The field dependence of q near the transition point is shown in the left inset of Fig. 2. At $h \approx 0.99$ this sheared dense lattice is replaced by the aligned lattice with the period $N = 2$ (structure *c*). At $h \approx 0.8$ this structure transforms back into the misaligned lattice with period $N = 1$ (structure *d*). At smaller fields, many lattice configurations compete for the ground state, as one can see more clear in the right inset. As a consequence transitions become more and more frequent at smaller field. At several fields (e.g., at $h \approx 0.19, 0.137, 0.105 \dots$) one or more lattice configurations have energies very close to the ground-state energy. We also note that there are several extended field ranges where in the ground state all layers homogeneously filled with vortices ($N = 1$) even in the region of dilute vortex lattice, e.g., for $0.115 < h < 0.17$, $0.21 < h < 0.38$. The layered structure favors such states. We also found

that the layered structure does not favor aligned structures with indices $(n, m) = (1, 0)$. Such structures do not realize in ground state for $2 < N < 7$.

In conclusions, we explored ground states of the Josephson vortex lattice in layered superconductors. With decreasing field a dense lattice transforms into a dilute lattice via an intermediate sheared dense lattice state. After that the lattice goes through sequence of states with different orientations and c-axis periods separated by first-order phase transitions. At low fields many lattice configurations compete for ground state and phase transitions become very frequent. As the energy differences between different states induced by the layered structure become tiny at small fields, external factors, such as interactions with boundaries or with correlated disorder, may play role in selection of ground states in real samples. Frequently such external interactions favor configurations aligned with the layered structure.

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